High-dimensional Problems in Finance and Economics

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Motivation

- Many problems in finance and economics are high dimensional.
  - Dynamic Optimization: Multiple kinds of capital stocks
  - DSGE: Multiple consumers/firms/countries
  - Games: Multiple players and states
  - Bayesian analyses compute high-dimensional integrals
  - Bootstrapping: analyze many n-dimensional samples from n data points
  - Simulation of large Markov processes - MCMC, Gibbs sampling, ACE
  - Parameter space searches to find robust conclusions
- Problems for both integration and approximation arise.

Curse of Dimensionality

- Many problems thus display the curse of dimensionality.
- The problem arises when you approximate a function on a grid.
- Use n points in each dimension.
- For a d-dimensional problem, you have n^d grid points.
- The dimensionality for a good approximation is thus limited.

What is the reaction?

- According to a giant in this field:
- Response I: Analyze silly models
  - Reduce heterogeneity in tastes, abilities, age, etc.
  - Assume no risk
  - Assume common information, beliefs, and learning rules
- Response II: Do bad math
- Response III: Do bad math when analyzing silly models

Bad news

- There is little appreciation for methods.
- A purely technical contribution usually does make its way into top journals.
- However you get a lot of attention if
  - you solve more general models and find new effects
  - if you create new models that couldn't be solved before.
- There is potentially a big benefit from cross-disciplinary research.

Outline

- I will talk about three ideas how economists deal with the curse of dimensionality.
- Two of the methods are used in my own work.
- The first method is grid-based and demonstrates the curse of dimensionality.
- Then I will introduce a model with many state variables (and thus dimensions)
- and two methods to solve it.

Introduction

- The first problem deals with estimation of economic models.
- Bayesian estimation involves integration of the posterior.
- We present a numerical quadrature approach for Bayesian estimation.
- We use sparse grids to deal with high dimensionality.
- It is an alternative to the use of simulations.

Bayesian Estimation

- Get data Y.
- Obtain likelihood function $\mathscr{L}(Y|\theta)$.
- Use prior information $p(\theta)$.
- Posterior: $p(\theta|Y) \sim \mathscr{L}(Y|\theta) \cdot p(\theta)$
- We then compute Bayesian estimators:

$$M = \int h(\theta)\mathscr{L}(Y|\theta)p(\theta)d\theta$$

Technique: MCMC

- MCMC (Markov-chain Monte Carlo) methods help to compute the integral.
- Create Markov-chain that has true distribution as stationary distribution.
- Pick starting point.
- Pick draws from each marginal distribution to get to the next step.

Technique: MCMC

- Metropolis-Hastings.
- Pick starting point.
- Draw sample ψ from proposal distribution $Q(\theta'_t|\theta_{t-1})$ (i.e. $Q(\theta'_t|\theta_{t-1}) \sim N(\theta_{t-1}, \Sigma)$).
- Accept draw only if $u < \frac{p(\psi)\mathscr{L}(Y|\psi)}{p(\theta_{t-1})\mathscr{L}(Y|\theta_{t-1})}$ where $u \sim U([0,1])$ otherwise $\theta_t = \theta_{t-1}$.
- Toss first N draws which is known as the "burn-in phase".

Introduction

- Main difficulty is dimensionality of the problem.
- Monte-Carlo methods converge at a rate *independent* of dimension — but slowly.
- Slow convergence puts restricitions on the set of models that can be estimated.
- We want a faster method.
- There is hope: the function is *very* smooth.

Introduction

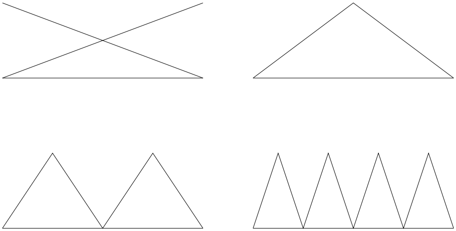
- Pseudo-random schemes give you convergence $O(N^{-\frac{1}{2}})$.
- Equidistributional sequences give convergence of order 1 for C^1-functions.
- There are non-simulation based methods with convergence of $O(N^{-k})$ for periodic C^k-functions.
- For very smooth functions, there is essentially no "curse of dimensionality".

Approximation

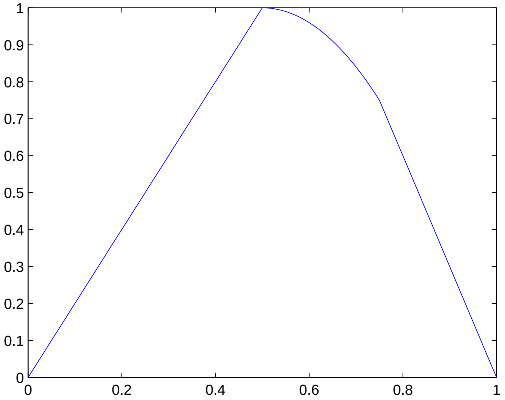
- Goal: Approximate function $f(x)$ with basis functions.
- We write the approximation $\hat{f}(x)$ as:

$$\hat{f}(x) = \sum_{(l,i)} u_{(l,i)} \phi_{(l,i)}(x) \approx f(x)$$

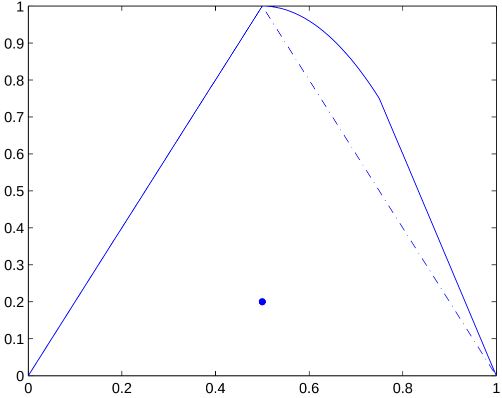
- Basis functions could for instance be:



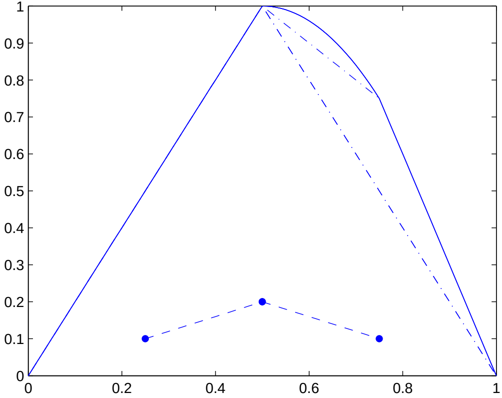
Approximation



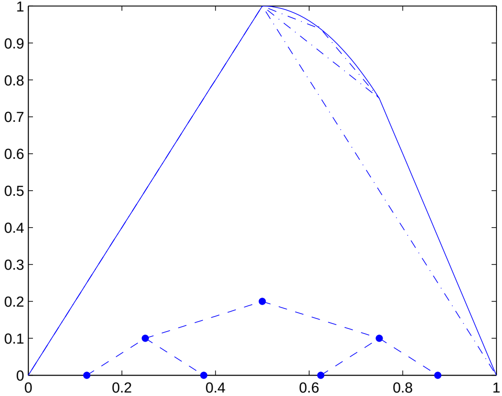
Approximation



Approximation



Approximation



Exponents

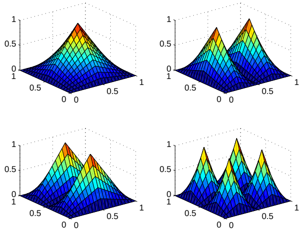
- Why can we truncate the sum at level l?
- Look at the approximation again.
- The absolute value of coefficients shrinks to zero.
- More precisely: $u_{(\mathbf{l},\mathbf{i})} \leq c \cdot 2^{-2 \cdot |\mathbf{l}|_1}$.

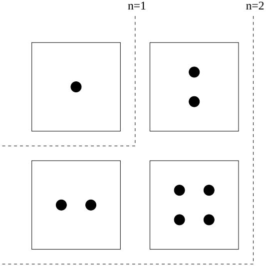
Sparse grids — Construction

- The goal is to generalize the one-dimensional approximation.
- This requires to specify:
  - the basis function
  - the grid

Sparse grids — Construction

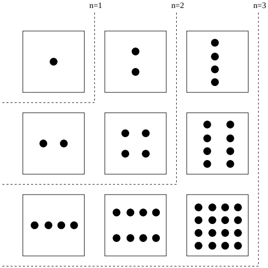
- Choice of basis functions and associated grid.

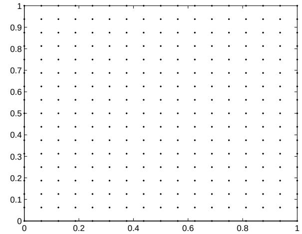




$$\phi_{(\mathbf{l},\mathbf{i})} = \prod_j \phi_{(l_j,i_j)}$$

Sparse grids — Construction

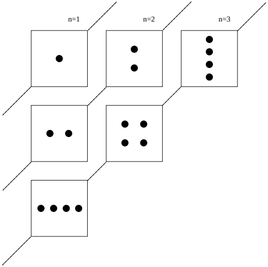


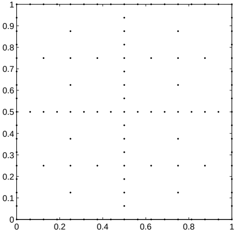


- "Curse of Dimensionality" for full grids — grid size grows at n^d.

Sparse grids — Construction

Sparse grids:





Approximation

- Note: We can now still write the approximation as

$$\hat{f}(\mathbf{x}) = \sum_{(\mathbf{l},\mathbf{i})} u_{(\mathbf{l},\mathbf{i})}\phi_{(\mathbf{l},\mathbf{i})}(\mathbf{x}) \approx f(\mathbf{x})$$

- where $\mathbf{l} = (l_1, \ldots, l_d)$, $\mathbf{i} = (i_1, \ldots, i_d)$
- $\phi_{(\mathbf{l},\mathbf{i})} = \prod_j \phi_{(l_j,i_j)}$
- It is a generalization of the 1-d concept.
- But: sparse grids come at a cost: you have to have bounded second mixed derivatives.

Features

	Full grids	Sparse grids
# grid points	n^d	$O(n \log(n)^{d-1})$
Accuracy		
L_2-error	$O(N^{-2})$	$O(N^{-2} \cdot \log(N)^{d-1})$
L_∞-error	$O(N^{-2})$	$O(N^{-2} \cdot \log(N)^{d-1})$

Integration and moments

- Having the approximation, we can now integrate
- Just sum up!

$$\int \sum_{(l,i)} u_{(l,i)} \phi_{(l,i)}(x) = \sum_{(l,i)} u_{(l,i)} \int \phi_{(l,i)}(x)$$

- How about computing moments?
- Use the same grid: since $\|f - \hat{f}\| < \delta$

$$\| \int x^2 (f(x) - \hat{f}(x)) dx \| < \delta \int x^2 dx$$

Bayesian Estimation

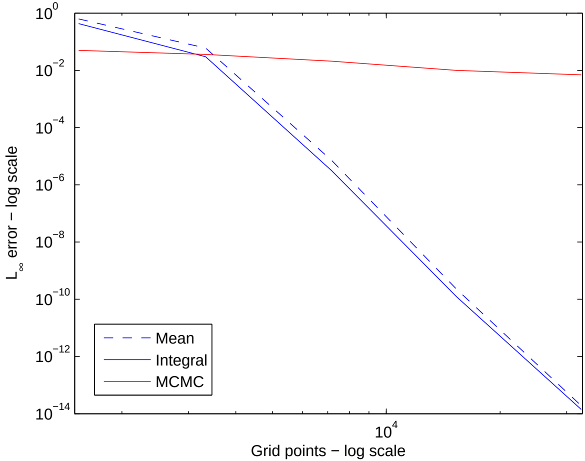
- Let's look at the problem again.
- Compute expected value of functions of θ

$$M = \int h(\theta)\mathscr{L}(Y|\theta)p(\theta)d\theta$$

- We can integrate directly.

Details

Numerical results



Result

- This solves three problems:
- 1. Method is fast.
- 2. Error estimation is accurate.
- 3. Get posterior to check for identification.

Motivation

- A large body of literature makes the simplifying assumption of a representative agent.
- In reality, people earn different incomes, have different talents, and hold different expectations.
- For this heterogeneity to impact equilibrium outcomes, asset markets need to be incomplete.
- In reality, we see this type of limited insurance.
- Uninsurable idiosyncratic risk produces heterogeneity across agents.

Difficulties

- With complete markets, agents can insure against idiosyncratic risk.
- As a result, we obtain aggregation.
- With incomplete markets, agents' choices are affected by their idiosyncratic shocks.
- Hence individual conditions differ across agents.
- The state space consists of one or more *distributions*.
- We have to deal with the curse of dimensionality.

Standard method

- Dynamic economies are typically solve by approximating the state space.
- Replace the law of motion by a linear function in aggregate state variables.
- You thus impose that prices are only a function of aggregate variables
- and *do not* depend on the distribution.
- This is an approximation which can produce poor solutions.

Modeling Strategy

Example: Standard Dynamic Stochastic General Equilibrium model

- **Households** Consumption-saving, capital-bond, subject to **uninsurable idiosyncratic labor income shocks**.
- **Representative Firm** Standard production.

Households (1/2)

- I households live in time periods $t = 0, \ldots, \infty$.
- Households choose consumption path and capital holdings to maximize

$$\max_{c_t^i, k_{t+1}^i, b_{t+1}^i} \sum_{t=0}^{\infty} \beta^t E_0 \left[u_c(c_t^i) - u_k(k_{t+1}^i, b_{t+1}^i) \right] \qquad i = 1, \ldots, I$$

subject to

$$c_t^i + k_{t+1}^i + b_{t+1}^i = (1 + r_t^k) k_t^i + (1 + r_t^b) b_t^i + w_t e^{\psi_t^i}.$$

- Labor income is subject to the individual shock ψ_t^i.

$$\psi_{t+1}^i = \rho_\psi^i \psi_t + \theta_{t+1}^i$$

Households (2/2)

$$u_k(k_{t+1}^i, b_{t+1}^i) = \nu_1 \frac{1}{(k_{t+1}^i + b_{t+1}^i - \underline{k})^2}$$

$$u_k(k_{t+1}^i, b_{t+1}^i) = \nu_1 \frac{1}{(k_{t+1}^i + b_{t+1}^i - \underline{k})^2} + \nu_2 (k_{t+1}^i - \bar{k})^2$$

$$u_k(k_{t+1}^i, b_{t+1}^i) = \nu_1 \frac{1}{(k_{t+1}^i + b_{t+1}^i - \underline{k})^2} + \nu_2 (k_{t+1}^i - \bar{k})^2 + \nu_3 (b_{t+1}^i)^2$$

$$u_k(k_{t+1}^i, b_{t+1}^i) = \nu_1 \frac{1}{(k_{t+1}^i + b_{t+1}^i - \underline{k})^2} + \nu_2 (k_{t+1}^i - \bar{k})^2 + \nu_3 (b_{t+1}^i)^2 + \nu_4 (k_{t+1}^i + b_{t+1}^i)$$

- The utility function $u_k(k_{t+1}, b_{t+1})$ has several components:
- Endogenous borrowing constraint.
- Defined steady-state distribution of capital holdings.
- Defined steady-state portfolio weights.
- Last term ensures that penalty function has global minimum at $\bar{k}$.

Firm

- Standard production sector.
- Perfect Competition, zero profits.
- Produce the final good according to $Y = f(K, L, z) = e^z K^\alpha L^{1-\alpha}$ where

$$z_{t+1} = \rho_z z_t + \eta_{t+1}$$

- Maximizing profits

$$\max_{K_t, L_t} Y_t - r_t^k K_t - w_t L_t$$

- returns first-order conditions:

$$r_t^k = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha}$$
$$w_t = (1-\alpha) e^{z_t} K_t^\alpha L_t^{-\alpha}$$

State space

- The state space of this economy is extremely large. For each agent, we need to keep track of
  - capital holdings
  - bond holdings
  - individual labor income shock
- Furthermore, we need to keep track of aggregate productivity.
- A more complicated model might have many more state variables for each agent.
- The challenge is to find the solution for economic behavior as a function of all state variables.
- In turn, next period's state variables are determined by the behavior.
- We need to solve a fixed-point problem on the state space.

Projection Methods

- One could build an approximation if the number of agents is relatively small.
- Therefore, one could use sparse grid methods.
- Still, there is a lot of space wasted.
- To see this, note that shocks are typically normally distributed in economics.
- A good area for approximation is not a cube but a sphere!

Outline

- The ergodic distribution can be at a different location in the state space than the deterministic version.
- To get a good approximation, one would like to approximate well around the ergodic mean.
- Stochastic simulation methods provide a way to solve the economy where it matters the most.
- We are looking for a consumption function for every single agent in the economy.

- To compute the equilibrium, we build an approximation to the choice functions.
- Therefore, pick a class of approximating functions (e.g. polynomials of degree n).
- We seek to find the best approximation in this class of functions.
- Once we have a guess, we can simulate the economy and check for accuracy.

Method

- Choose a flexible class of functional forms $\psi(s; b)$.
- b parameterizes the approximating function.
- To set up, re-write the optimality conditions for all agents in the form

$$k_{t+1}(s_t) = E[\beta \frac{u'(c_{t+1})}{u'(c_t)}(1 + e^{z_{t+1}} f'(k_{t+1}, l_{t+1}))]$$

Algorithm

- Pick an initial set of parameters b_0.
- Choose an intial point in the state space s_0.
- In iteration i:
- Simulate the economy for T periods using the policy rule $\psi(s, b_i)$

Algorithm

- For each t, define the optimal choice under the rule $k_{t+1} = \psi(s; b)$:

$$y_t = E[\beta \frac{u'(c_{t+1})}{u'(c_t)}(1 + e^{z_{t+1}} f'(k_{t+1}, l_{t+1}))]$$

- Find the best approximation b^* by solving

$$\min_b \|\varepsilon_{t=0}^{T-1}\| \tag{1}$$

- for some norm $\| \cdot \|$.
- Updating:

$$b^{i+1} = (1 - \xi) b^i + \xi b^*$$

- Iterate until convergence.

Discussion

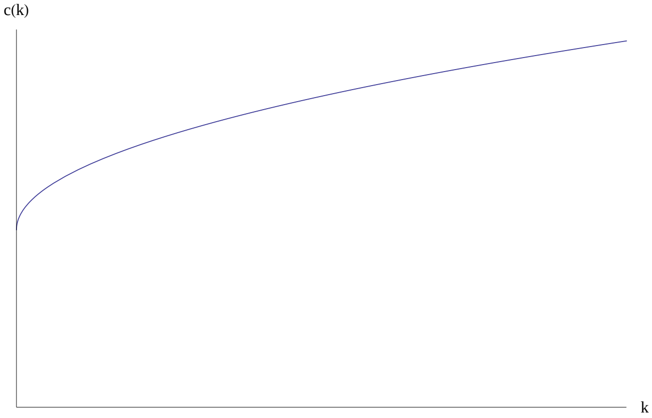
- Good method for medium-scale economies.
- Easy to program
- Typically each iteration can be solved quickly
- but it can potentially take many iterations.

Back to first principles

- Usually little is known about the solution (or even existence) for incomplete market economies.
- We focus on a broad class of models with aggregate and uninsurable idiosyncratic risk.
- **What do we know?**
  1. Solution at the deterministic steady-state.
  2. Solution to the deterministic case exists.
  3. Equilibrium conditions are typically "well-behaved".
- **How can we use this information to construct a solution to the stochastic problem?**

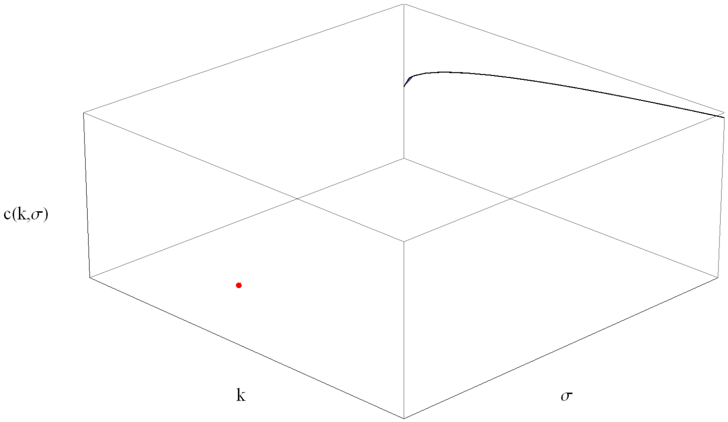
Graphical illustration (1/3)

- Desired solution.



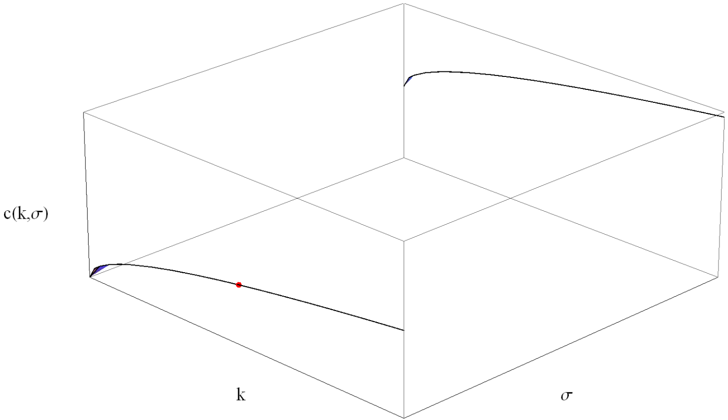
Graphical illustration (2/3)

- We embed this solution in a larger space.



Graphical illustration (3/3)

- We start the solution at the no-risk case.



Key idea

- Using perturbation methods turns out to be particularly convenient.
- The key observations are:
  1. At the deterministic steady-state, all agents are identical.
  2. Expansions around this steady-state are symmetrical:
  3. Only need to expand optimality conditions for one agent.
  4. Only need to expand in state variables for two agents (first-order).

Starting point

- Equilibrium conditions in the form

$$E_t \left[g^1(\mathbf{X}_t, \mathbf{z}_t, \mathbf{C}_t, \mathbf{P}_t, \mathbf{X}_{t+1}, \mathbf{z}_{t+1}, \mathbf{C}_{t+1}, \mathbf{P}_{t+1})\right] = 0$$
$$\mathbf{X}_{t+1} = g^2(\mathbf{X}_t, \mathbf{z}_t, \mathbf{C}_t, \mathbf{P}_t)$$

- F contains:
  - first-order conditions of all agents (Euler equations)
  - Law of motions
  - Market clearing conditions
- First-order conditions are identical for all agents.
- Thus: expand only one optimality condition.

Example

$$
\begin{pmatrix} x_1^1 & x_1^2 & \dots & x_1^N \\ & \vdots & \vdots & \\ x_I^1 & x_I^2 & \dots & x_I^N \end{pmatrix}, \begin{pmatrix} z_1 \\ \vdots \\ z_Z \end{pmatrix}, \begin{pmatrix} \mathbf{c}_1^i \left(\begin{pmatrix} x_1^1 & x_1^2 & \dots x_1^N \\ & \vdots & \vdots \\ x_I^1 & x_I^2 & \dots x_I^N \end{pmatrix}, \begin{pmatrix} z_1 \\ \vdots \\ z_Z \end{pmatrix}, \sigma \right) \\ \vdots \\ \mathbf{c}_C^i \left(\begin{pmatrix} x_1^1 & x_1^2 & \dots x_1^N \\ & \vdots & \vdots \\ x_I^1 & x_I^2 & \dots x_I^N \end{pmatrix}, \begin{pmatrix} z_1 \\ \vdots \\ z_Z \end{pmatrix}, \sigma \right) \end{pmatrix}
$$

In our example:

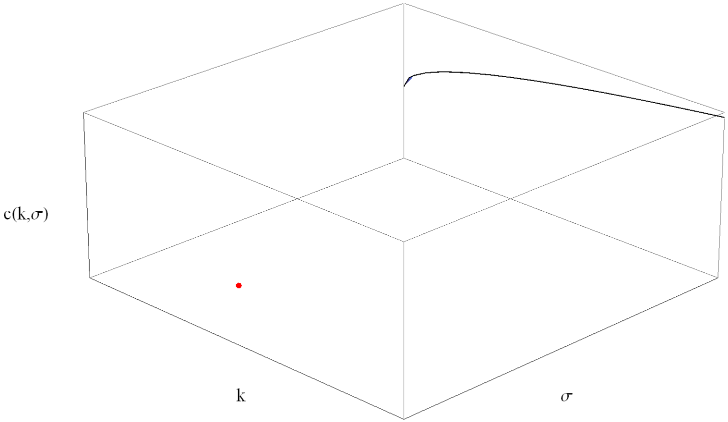
$$
\begin{pmatrix} x_1^1 & x_1^2 & \dots & x_1^N \\ & \vdots & \vdots & \\ x_I^1 & x_I^2 & \dots & x_I^N \end{pmatrix} = \begin{pmatrix} k_1^1 & b_1^2 & \psi_1 \\ \vdots & & \vdots \\ k_I^1 & b_I^2 & \psi_I \end{pmatrix}
$$

Deterministic steady-state

- Deterministic steady-state is defined.
- It features a degenerate distribution of capital.
- Fixing this distribution is important to avoid unit roots.
- Once we move to a stochastic economy, we no longer need to fix the portfolio exogenously.

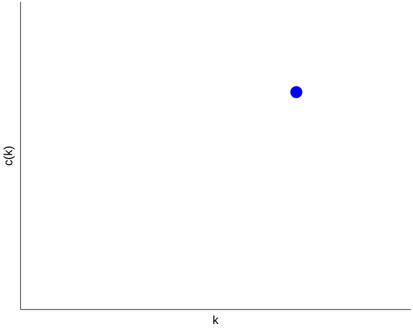
Deterministic steady-state

- We start the solution at the no-risk case.



Deterministic steady-state

- Deterministic steady-state.



Perturbation methods: Example

We want to solve the equation

$$F(y(x), x) = 0$$

$$\frac{1}{y(x)} - x + \frac{x^2 + x - 1}{y(x)} = 0$$

to which the solution is $y(x) = x + 1$.
Expand this equation around $x_0 = 1$:

$$\frac{1}{y_0} - x_0 + \frac{x_0^2 + x_0 - 1}{y_0} = 0$$

Expand this equation around $x_0 = 1$:

$$\frac{1}{y_0} - 1 + \frac{1}{y_0} = 0$$

Expand this equation around $x_0 = 1$:

$$y_0 = 2$$

First-order term at $x_0 = 1$:

$$dF(y(x), x)\Big|$$

- Using perturbation methods, we build a *higher-order approximation* around the deterministic steady-state.
- Perturbation methods build a Taylor series approximation of the policy functions and prices around the point of expansion.

$$\mathbf{C}^i_\varpi = \sum_{o=1}^{\infty} \sum_{\|\mathbf{l}\|+|\mathbf{j}|+|\mathbf{k}|=o} \frac{1}{\mathbf{l}! \cdot \mathbf{j}! \cdot \mathbf{k}!} \frac{\partial^o \mathbf{C}^i_\varpi}{\partial \mathbf{X}^\mathbf{l} \partial \mathbf{z}^\mathbf{j} \partial \sigma^\mathbf{k}}\bigg|_{(\mathbf{X}^0, \mathbf{z}^0, 0)} \langle\langle (\mathbf{X} - \mathbf{X}^0)^\mathbf{l} \rangle\rangle \langle (\mathbf{z} - \mathbf{z}^0)^\mathbf{j} \rangle \cdot \sigma^k$$

$$\mathbf{C}^i_\varpi = \sum_{o=1}^{\infty} \sum_{\|\mathbf{l}\|+|\mathbf{j}|+|\mathbf{k}|=o} \frac{1}{\mathbf{l}! \cdot \mathbf{j}! \cdot \mathbf{k}!} \frac{\partial^o \mathbf{C}^i_\varpi}{\partial \mathbf{X}^\mathbf{l} \partial \mathbf{z}^\mathbf{j} \partial \sigma^\mathbf{k}}\bigg|_{(\mathbf{X}^0, \mathbf{z}^0, 0)} \langle\langle (\mathbf{X} - \mathbf{X}^0)^\mathbf{l} \rangle\rangle \langle (\mathbf{z} - \mathbf{z}^0)^\mathbf{j} \rangle \cdot \sigma^k$$

Higher-order approximation (1/4)

- Obtain derivatives as a solution to

$$g_i^1(\mathbf{X}_t, \mathbf{z}_t, \mathbf{C}_t, \mathbf{P}_t, \mathbf{X}_{t+1}, \mathbf{z}_{t+1}, \mathbf{C}_{t+1}, \mathbf{P}_{t+1}) = 0$$

$$\left.\frac{dg_i^1(\mathbf{X}_t, \mathbf{z}_t, \mathbf{C}_t, \mathbf{P}_t, \mathbf{X}_{t+1}, \mathbf{z}_{t+1}, \mathbf{C}_{t+1}, \mathbf{P}_{t+1})}{dx_1^1}\right|_{(\mathbf{X}^0, \mathbf{z}^0, 0)} = 0$$

$$\frac{\partial g_i^1}{\partial x_1^1} + \frac{\partial g_i^1}{\partial \mathbf{C}_t}\frac{\partial \mathbf{C}_t}{\partial x_1^1} + \frac{\partial g_i^1}{\partial \mathbf{P}_t}\frac{\partial \mathbf{P}_t}{\partial x_1^1} + \frac{\partial g_i^1}{\partial \mathbf{X}_{t+1}}\frac{\partial \mathbf{X}_{t+1}}{\partial x_1^1} + \frac{\partial g_i^1}{\partial \mathbf{z}_{t+1}}\frac{\partial \mathbf{z}_{t+1}}{\partial x_1^1} + \frac{\partial g_i^1}{\partial \mathbf{C}_{t+1}}\frac{\partial \mathbf{C}_{t+1}}{\partial x_1^1} + \frac{\partial g_i^1}{\partial \mathbf{P}_{t+1}}\frac{\partial \mathbf{P}_{t+1}}{\partial x_1^1} = 0$$

- **Key steps**
  - Only expand one optimality condition.
  - Expand only in few state variables.

Higher-order approximation (2/4)

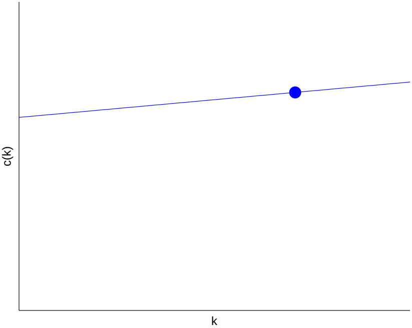
- We exploit the symmetry of the expansion.
- In a first-order, we only need to expand in two directions for a given state variable:
  - with respect to the person's own state and
  - with respect to any other person's state.
- Higher-order expansions become slightly more complicated.

Higher-order approximation (3/4)

- First-order approximation corresponds to linearization.
- Solving the system for linear coefficients can be challenging.
- There are potentially multiple solutions (Riccati-type equations).
- But the system is typically polynomial and we resort to standard methods.

Linearization

- Build linear approximation.

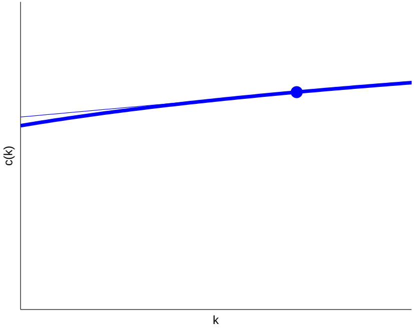


Higher-order approximation (4/4)

- The number of coefficients grows but it remains manageable.
- Expansions of order higher than one lead to linear systems.
- Higher orders are important because
  - Effects of heterogeneity
  - Effects of stochasticity

Higher-order approximation

- Build higher-order derivatives in state variables

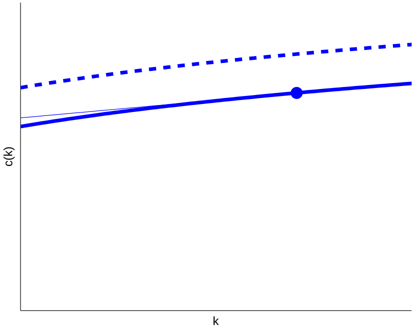


Uncertainty

- So far: approximate solution to the deterministic case.
- Last step: move to the stochastic economy.
- Therefore, build expansion with respect to standard deviation of shocks.
- If there are multiple shocks, we scale them proportionately.
- With higher derivatives, we get a mean-variance-skewness theory.

Uncertainty

- Build derivatives with respect to standard deviation of shocks.
- Thereby, we move from the deterministic to the stochastic economy.



Uncertainty

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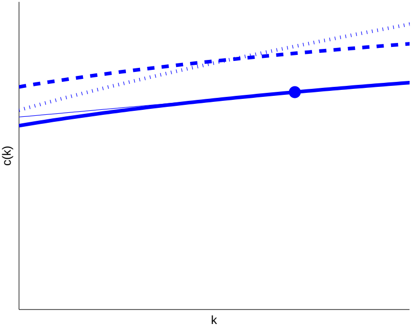


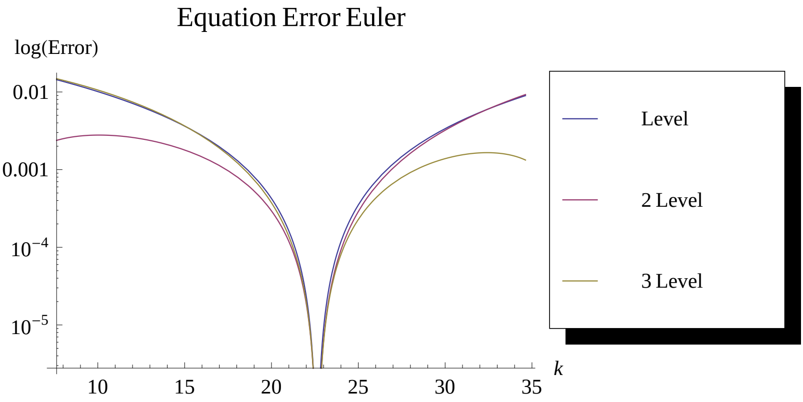
Range of applications (2/2)

- Dynamic Programming problems
- We can solve a planner's problem
- General form of the Dynamic Program

$$V(\mathbf{X}_t, \mathbf{z}_t, \sigma) = u(\mathbf{C}_t) + \beta E_t[V(\mathbf{X}_{t+1}, \mathbf{z}_{t+1}, \sigma)]$$

Euler Equation Error

- We see convergence.



Analysis of the SDF

- We analyze the "aggregate" stochastic discount factor

$$\begin{aligned}\sum_{i=1}^{I} \beta \frac{u'(c_{t+1}^i)}{u'(c_t^i)} =& c(\mathbf{X}_t, \mathbf{z}_t, \mathbf{k}_{t+1}, \mathbf{b}_{t+1}) \\ & + c_z^{(1)} e^{z_{t+1}} + c_z^{(2)} \text{var}(e^{z_{t+1}}) + \ldots \\ & + c_\psi^{(2)} \text{var}(\psi^i) + \ldots\end{aligned}$$

- The expansion can be used to price assets.

Impact of heterogeneity

- Compared with the representative agent counterpart, the steady-state level of capital in the stochastic economy is higher.
- There is more heterogeneity and thus more reason to build up precautionary savings.
- The introduction of a bond market mitigates the effects on the steady-state.
- The second-order term consists of two parts:
  - The variance of the distribution of capital
  - The comovement of individual with aggregate capital.
- This tells us how to design solution methods. Simply adding more moments to the state space is not sufficient.

Example: Lucas tree

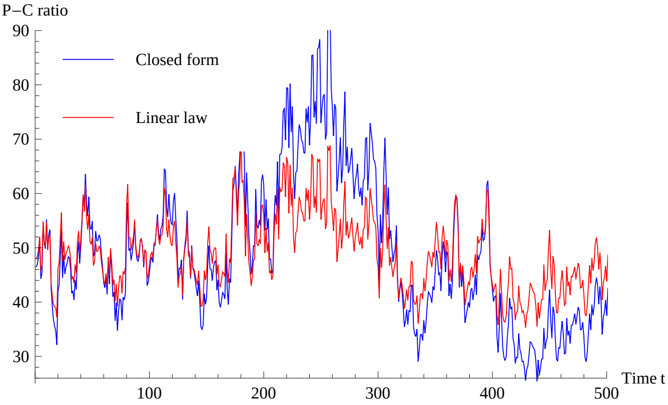
- Endowment economy with preference shocks.
- Preferences are given by $U = E_0 \left[\sum \beta^j A_j B_j \frac{C_j^{1-\gamma}}{1-\gamma} \right]$
- The stochastic processes are

$$
\begin{aligned}
\log(C_{t+1}) &= \log(C_t) + \varepsilon_{t+1},\ \varepsilon_{t+1} \sim N(\mu, \sigma_\varepsilon^2) \\
\log(A_{t+1}) &= \rho_A \log(A_t) + \sigma_A \eta_{t+1},\ \eta_{t+1} \sim (0,1) \\
\log(B_{t+1}) &= \rho_B \log(B_t) + \sigma_B \eta_{t+1},\ \eta_{t+1}
\end{aligned}
$$

- Claim to tree trades at P_t solvable in closed form.
- Linear law: $\frac{P_{t+1}}{C_{t+1}} = \alpha_0 + \alpha_1 \frac{P_t}{C_t} + \alpha_2 \eta_{t+1}$
- We show example for a particular parameterization.

Results

- Here are the PD-ratios for this example.
- The R^2 diagnostic leads to more than 98% for the linear law.



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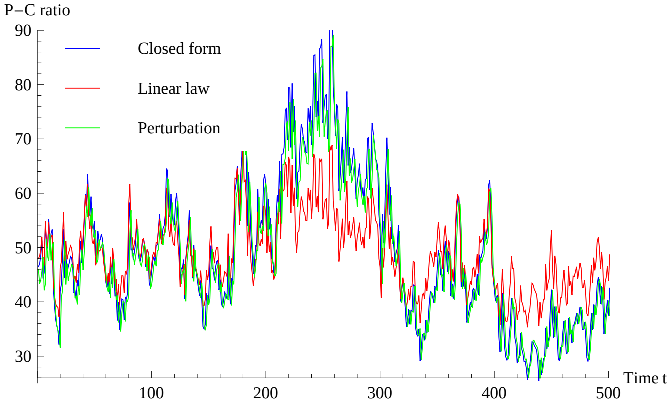

Further questions?

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