

# A Network Model of Sorting Effects in Education

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Prepared for a course on Methods of Computational Statistics  
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# Outline

- 1 Introduction
  - The Motivation
  - The Question
  - Preview of Main Results
- 2 The Model
  - The Tools
  - An Intergenerational Model
- 3 The Results
  - Constant Population
  - With Population Growth or Decay

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# The Motivation

What drives trends in Educational or Human Capital Inequality?

Dynastic Inequality (*Material Constraints*)

- Basis for equal opportunity and redistributive policies.

Beliefs and Values (*Non-Material Constraints*)

- Basis for a 'seek your own path' attitude.

The Usual Suspects

- Opportunism, discrimination, corruption, etc.
- Free Market Healthcare and Education.

Basic Decision-Making (*Given Constraints*) → Stratification

- Residential, Peer and Marriage choices → Disjoint Politics, Unequal Tax Districts, Segregation in Schooling by Income

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What drives trends in Educational or Human Capital Inequality?

*Example 1:* California Supreme Court ruling (Serrano vs. Priest II, 1976) declared unconstitutional any financing that correlates districts' school expenditures with taxable wealth, such as property or income [Benabou, 1996].

*Example 2:*

TABLE I  
TOTAL SPENDING (\$) PER STUDENT FOR THE 1986-1987 ACADEMIC YEAR  
(PRIMARY AND SECONDARY SCHOOLING)

New Jersey Area		Detroit Area		New York Area	
MontClair	6,442	Bloomfield Hills	6,976	Great Neck	12,868
Parsippany-Troy Hills	6,229	Birmingham	6,668	White Plains	11,045
Cherry Hill	5,695	Grosse Pointe	5,705	Syosset	9,125
Jersey City	5,656	Royal Oak	5,172	New Rochelle	7,970
East Orange	5,005	Pontiac	4,553	Hampstead	7,462
Camden	4,871	Detroit	3,854	Baldwin	7,251
Paterson	4,708	East Detroit	3,740	Levittown	7,210
Gloucester	3,814	Highland Park	3,105	New York City	6,433
Pemberton	3,668	Dearborn	2,684	Mount Vernon	6,328

Source: 1987 Census of Governments.

# The Motivation

## Stratification as a Self-Organized Social Structure

- Birds of a Feather Flock Together
- Synonyms: Stratified Homophily, Assortative Matching, Sorting, Preferential Attachment, Agglomeration

## Normative Ambiguity of Stratification

- Segregation and Disconnect
- Untapped Positive and Contained Negative Externalities
- Benefits of Specialization

## Sorting as a Tool to Describe Stratification

- Civil Union Network
- Peer Effects in Education



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# A Comment on the Literature and a Language Disclaimer

## Public vs. Private Education

- Glomm and Ravikumar (1992)

## Communities and the Quality of Public Education

- Fernandez and Rogerson (1996)

## Sorting and Long-Run Inequality

- Fernandez and Rogerson (2001)

## On Stratification and the Community Structure of School Finance

- Benabou (1996)

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# Preview of the Model

- States: Individuals enter each period as Skilled  $S_t$  or Unskilled  $U_t$ .
- Agents:  $(S_t, U_t)$  populations meet with matching parameter  $\theta$  and reproduce at rates  $f_1, f_2, f_3$ .
- Types: Pairings result in household types  $(h, m, l)$ .
- Neighborhoods: Kids from different households interact in neighborhoods of mixed, sorted, or intermediate 'disorder'  $\phi$ .
- Information: Peers influence each other with weights:  $w_1, w_2, w_3$ .
- Actions: Based on parents' encouragement and peer effects, children decide to go to college or not.
- Outcomes: Skilled population increases or decreases  $(S_{t+1}, U_{t+1})$ .

# The Question

For a given set of parameters,

- Fertility Rates,  $f_j \forall j$
- Level of Marital Sorting,  $\theta$
- Interaction Strengths of Peer Network,  $w_i \forall i$

Is there a critical level of Educational Mixing  $\phi$  that will nudge the system into the equilibrium characterized by lower population growth and higher aggregate human capital?

If so, how might housing and educational policy be designed to aim at this target?

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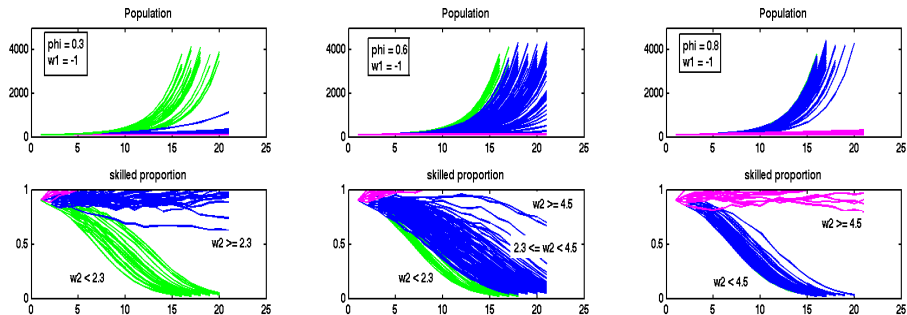
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# Preview of Main Results

Increasing the disorder in the peer network, I find that the skilled equilibrium is attained for increasingly weaker interaction strengths.



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# Assortative Matching: Sorting and Mixing

Single-Crossing Preferences  $\rightarrow$  Partially or Fully Separating Eqm.

- Individuals have an incentive to sort, despite social costs.
  - Preferences over taxes, housing prices, school quality.
  - Become stratified over income, ability, human capital.
- Multi-community equilibria grouped by type.
  - High, middle, low and shades inbetween.
- Preference structure guarantees majority voting.

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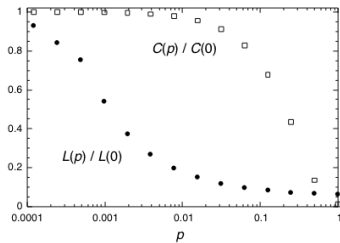
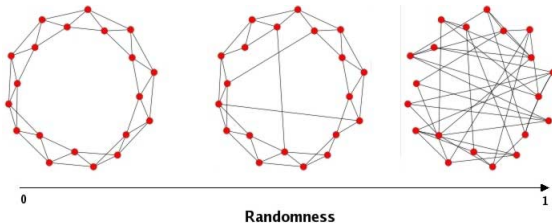
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# Small World Networks



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# An Intergenerational Model

## The Timeline:

- ① Skilled and unskilled individuals ( $S_t, U_t$ ) pair up to form households of types  $(h, m, l)$ .
- ② Depending on their type, households make decisions with regard to:
  - ① Fertility
  - ② Children's Education
- ③ Children go to school and interact with other children.
- ④ Peers influence a child's decision to pursue higher education and become skilled or not ( $S_{t+1}, U_{t+1}$ ).

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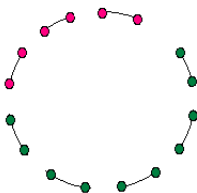
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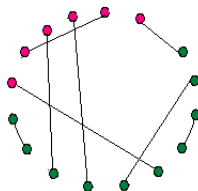
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# An Intergenerational Model

## 1. The Civil Union Network ( $S_t, U_t$ )



theta = 1



theta = 0

Network topologies can be encoded in matrices:

$$\begin{pmatrix} S & 1 & 0 & 0 & 0 & 0 \\ 1 & S & 0 & 0 & 0 & 0 \\ 0 & 0 & U & 1 & 0 & 0 \\ 0 & 0 & 1 & U & 0 & 0 \\ 0 & 0 & 0 & 0 & U & 1 \\ 0 & 0 & 0 & 0 & 1 & U \end{pmatrix}$$

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# An Intergenerational Model

## 2. Household Decisions ( $h, m, l$ )

### (a) Fertility

-Fertility varies across family types.

### (b) Child's Education

-High types insist all children go to college.

-Medium types may encourage kids to attend college.

-Low types do not encourage children to go to college.

Two competing effects → Can we anticipate a balance point that sustains a steady ratio of skilled to unskilled parents?

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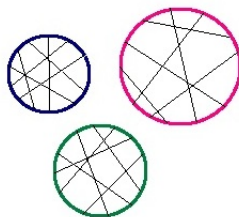
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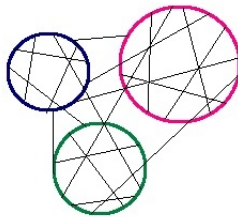
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## 3. Interaction of Kids in School

- Interaction Topology:



$\phi = 1$



$\phi = 0$

- Interaction Dynamic:

- Epidemiological Dynamic: SI model

- Due to household structure, cannot make a mean field approximation in the random graph extreme.

# An Intergenerational Model

## 4. Peer Effects in Early Education

Two college-bound kids reinforce with weight  $w_1 > 0$ .

Two non-college-bound kids reinforce with weight  $w_3 < 0$ .

College-bound kid influences a non-college-bound kid,  $w_2 > 0$ .

- Based on interactions, kids choose to become skilled  $S_{t+1}$  or unskilled  $U_{t+1}$ .
- The fraction of new skilled individuals is the ratio:  $\beta_{t+1} = \frac{S_{t+1}}{N_{t+1}}$
- Alternatively,  $w_2 < 0$  would indicate an opposite influence, such that not attending college has the dominant marginal effect.
- Alternatively, interactions weights should be stochastic.

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Is there a critical level of Educational Mixing  $\phi$  that will nudge the system into the higher equilibrium, characterized by lower population growth and higher aggregate human capital?

# Empirical Calibration of Parameters

Try to tie the model to reality by fitting parameters to the data.

- Fernandez and Rogerson (2001) discuss calibration:
  - Fertility:  $f_1=1.84$ ,  $f_2 = 1.90$ ,  $f_3 = 2.26$
  - Marital sorting:  $\theta = 0.6$
- Need a better theory for peer interactions ( $w_1, w_2, w_3$ ).
  - How do kids influence each other?
  - Retro-survey kids on impact of past peer networks.
- Parameters vary over city and through time  $\Rightarrow$  results are location specific (not externally valid).

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Incorporate distributions we have spoken about in class:

- Marital Sorting and/or Educational Sorting
  - Characterize disorder with an alternative rewiring mechanism.
  - Use a chinese restaurant process to rewire.
- Parental Influence on Education Decision
  - Parents choose to send  $x$  out of  $n$  kids to school  $\Rightarrow$  binomial
  - Endogenize parental influence
    - I.e. parents know something about the labor market and advise kids accordingly.
- Peer Interaction Dynamic
  - Survey adults on peer influences as kids.
  - Fit weights to one of our distributions.

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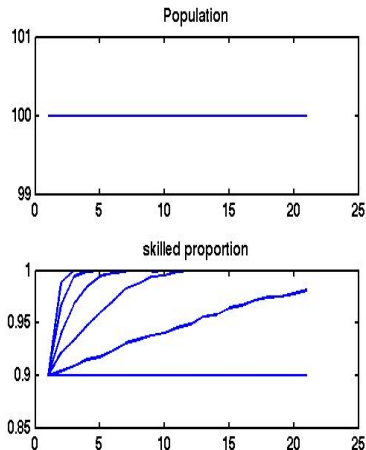
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# Same Fertility Rates, $f_j = 2 \forall j$



Benchmark Case:

Different levels of educational sorting,  $\phi = 0, 0.2, 0.4, 0.6, 0.8, 1$ , give rise to variation in decay rates.

Each curve is an average of 30 realizations.

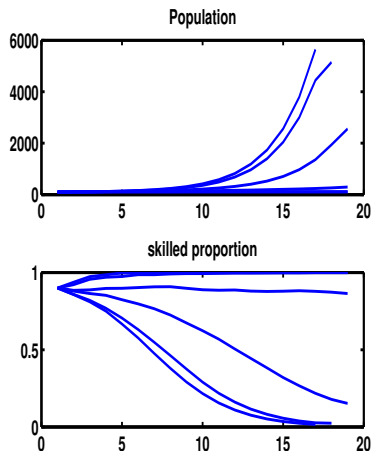
With constant fertility, system always goes to high equilibrium.

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# Varied Fertility Rates, $f_1 = f_2 = 2$ , $f_3 = 3$



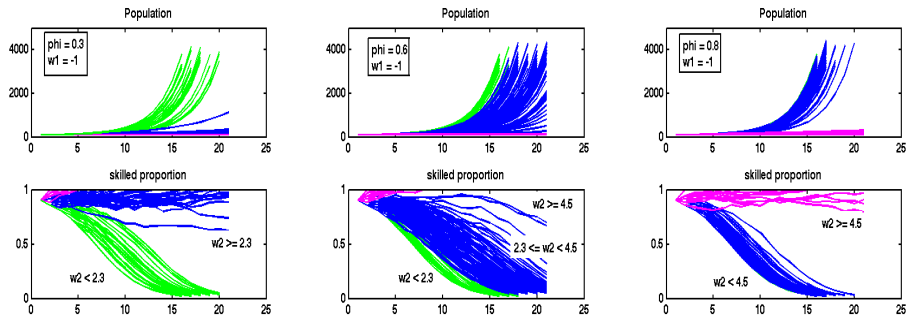
Full Marital Sorting,  $\theta = 1$

For low levels of educational sorting  $\phi = 0, 0.2$ , the system quickly converges to the fully skilled eqm.

For high levels of sorting  $\phi = 0.8, 1$ , the unskilled equilibrium is attained.

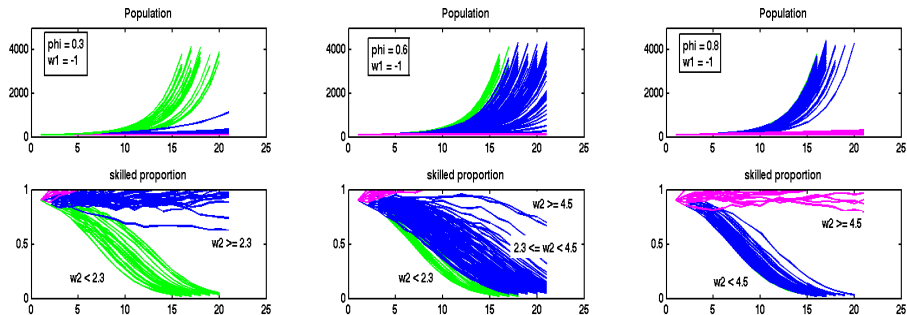
For an intermediate level of sorting  $\phi = 0.4$ , a constant ratio of skilled to unskilled is maintained *on average*, but is not necessarily stable.

# Varied Fertility Rates, $f_1 = f_2 = 2$ , $f_3 = 3$



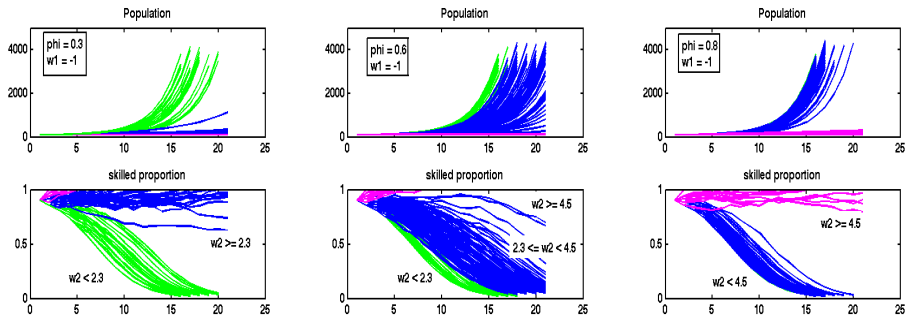
- Full Marital Sorting,  $\theta = 1 \Rightarrow \nexists$  middle type household.
- For US in 1968, PSID gave  $\theta = 0.6$ .
- Higher sorting in developing countries.

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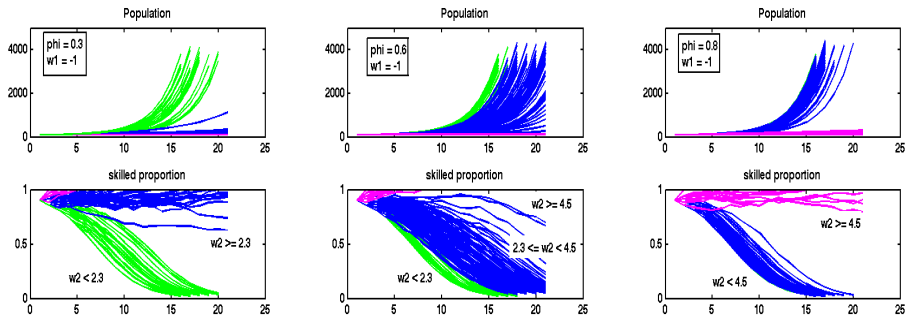
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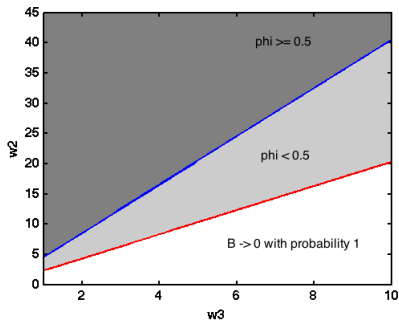
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# Parameter Phase Space

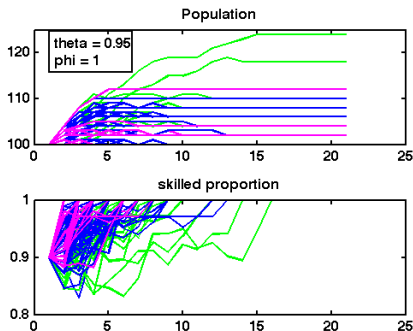


To answer The Question posed:

- For a given set of interaction weights,  $\exists$  a critical  $\phi$ , below which  $\beta \rightarrow 0$  with probability one.

Some Marital Mixing,  $\theta < 1$ 

If  $w_2 > 0$ ,  $\beta \rightarrow 1 \forall \phi$ .



Suggests that interaction weights  $w_1, w_2, w_3$  should be drawn from a distribution with support on the real line (positive and negative).

# Summary

- We discussed stratification over skill induced by marriage and residential/schooling decisions.
- We found non-interesting results for parameter values that are empirically calibrated to US 1968 PSID data.
- For parameters pertaining to a different data set, we find an interesting nonlinear effect.
- Future Research
  - Draw interaction strengths from a distribution over the real number line.
  - How can the interaction dynamic be more realistically constructed?
  - To what extent can  $\phi$  and other parameters be affected by policy and campaigning?
  - Include parents' beliefs and learning in the model.



# Discussion

If basic decision-making is a source of stratification, is there something that can be done to counterbalance the adverse effects?

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- Inform families of the benefits of a diverse educational experience.
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